RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. SECOND SEMESTER EXAMINATION, MAY-JUNE 2013

FIRST YEAR

Date : 20/05/2013 **Physics (Honours)**

Time : 11am – 3pm Paper : II Full Marks : 75

[Use separate answer book for each group]

Group - A

Sec - I

Answer **any three** questions out of question no. 1 to 5

1. a) Find the radial and transverse components of the acceleration of a moving particle in spherical polar coordinates. (4)

   b) Prove that for a system of particles, the angular momentum about a point is the sum of the angular momentum of a particle of equal mass placed at the centre of mass and the angular momentum of the particles about the centre of mass. (2)

   c) Two inertial reference frames S and S' have a relative velocity $\vec{V}$. Two particles of masses m_1 and m_2 undergo an elastic collision. Show that if the total momentum of the particles before and after collision is conserved in S, they are also conserved in S'. (4)

2. A particle of unit mass is projected vertically upwards in the gravitational field of earth, with initial velocity V_o. The medium offers a resistance to motion proportional to the square of the instantaneous speed v, i.e, $R = -kv^2$ (k>0).

   a) Write down the equation of the motion and calculate the time of rise to the maximum height H. (1+2)

   b) Show that the maximum height reached is $H=\frac{c^2}{2g}\log\left[1+\frac{{v_o}^2}{c^2}\right]$, where $c^2=\frac{g}{k}$. (3)

   c) Show that the velocity of the particle when it falls back to the ground is given by $\frac{cv_o}{\sqrt{{v_o}^2+c^2}}$. (4)

3. a) Consider a frame R rotating uniformly relative to an inertial frame S, having the same origin. If the equation of motion of a particle w.r.t S is given by $\vec{F}=m\ddot{\vec{r}}$, what is its equation of motion w.r.t frame R? Interpret the various terms present in this equation. (5)

   b) A body is falling down from a height 'Z' on the earth's surface at a place of latitude λ in the northern hemisphere. Show that the body is deflected by an amount $\frac{2\sqrt{2}}{3}\omega\sqrt{\frac{Z^3}{g}}\cos\lambda$. The symbols have their usual meaning. (5)

4. a) Show that for a particle moving in a central force, the total energy and momentum are constants of motion. Hence show that the path of the particle is confined to a plane. (3)

   b) For a particle moving in a force field $f(r)=-\frac{k}{r^2}\quad(k>0)$, show that the total energy E is given by $E=\frac{1}{2}mh^2\left[u^2+\left(\frac{du}{d\theta}\right)^2\right]-ku$, where $u=\frac{1}{r}$ and h = twice the areal velocity. (3)

   c) Use (b) to obtain the differential equation for the orbit $u = u(\theta)$. (2)

   d) A particle moves in a spiral orbit, $r=a\theta$. If θ increases linearly with time t, is the force a central force? If not, find how should θ vary with t, for a central force. (2)

5. a) A rigid body is rotating about an arbitrary axis with an angular velocity $\vec{\omega}$. Find an expression for its angular momentum and rotational kinetic energy in terms of moment of inertia. (4)

b) What are the principal axes and principal moments of inertia at a point? (2)

c) Find the moments and products of inertia of a square plate of side 'a' about x, y, z axes, x and y being taken as the adjacent sides of the plate and z axis perpendicular to its plane. Find also the principal moments of inertia in this case. (4)

Sec - II

Answer **any two** questions out of question no. 6 to 9

6. a) A uniform cylinder of length L and radius R is fixed at one end and subjected to a torque at the other end. Find the torsional rigidity, given that the rigidity modulus of the material is 'n'. (5)

b) Compare the loads required to produce equal depressions at the end for two beams made of the same material and having the same length and weight with only difference that one is of circular cross-section while the cross-section of the other is square. (3)

c) State the equation of continuity in fluid motion. What happens when the fluid is incompressible? (2)

7. a) A homogeneous sphere of mass M has a radius R. Consider a point P at a distance x from the centre O. Show that gravitational potential at P is $V_p = -\frac{1}{2}\frac{GM}{R^2}(3R^2 - x^2), \quad x \le R.$

Draw graphs showing the variation of the intensity and potential vs x in the range $0 \le x \le R.$ (4+2)

b) The density of the material within a spherical body varies inversely as the distance from the centre. Show that the gravitational field inside the sphere is the same everywhere. (4)

8. a) What is a Newtonian fluid? Give an example of a non-Newtonian fluid. (2)

b) Define Reynold's number. (2)

c) Two soap bubbles of equal volume combines to form a single bubble. As a result, if the change in volume is V and the change in surface area is S, show that $PV = \frac{4}{3}ST$, where P is the atmospheric pressure and T is the surface tension of the soap solution. (4)

d) A vessel has a hole of radius r at its bottom. Show that a liquid kept inside the vessel will come out of the vessel if its depth exceeds $h = \frac{2T}{\rho g r}$, where T is the surface tension of the liquid, and ρ is its density. (2)

9. a) Set up Euler's equation in hydrodynamics for an incompressible fluid. (3)

b) From the above equation deduce Bernoulli's theorem for steady streamline flow. (4)

c) State Stoke's law in connection with the viscous drag experienced by a moving spherical body. Hence find the terminal velocity of a rain drop with radius 10^{-5} metre falling through air. (co-efficient of viscosity of air = 1.8×10^{-5} decapoise). (1+2)

Group - B

Answer **any two** questions out of question no. 10 to 13

10. a) For a damped oscillator with damping force proportional to its velocity,

i) Solve the equation of motion for the case of critical damping. (4)

ii) Show that for such a motion if the initial displacement is zero and the initial velocity is non-zero, then the time at which the oscillator stops is independent of the initial velocity. (2)

b) Show that the average energy density contained in a stationary wave is twice that for a progressive wave. (4)

11. a) Find an expression for the velocity of plane longitudinal wave in fluid media mentioning the assumptions made. (4+2)

b) If C be the speed of sound in a gaseous medium $(\gamma = 1.4)$, show that the rms speed V of the gas molecules is $V = \left(\frac{15}{7}\right)^{\frac{1}{2}} C$. (4)

12. a) Starting from the equation of a simple harmonic oscillator with damping (proportional to velocity) and an external periodic force, deduce the condition of velocity resonance. Hence explain sharpness of resonance. (6+2)

b) Define group velocity and phase velocity. (2)

13. a) Derive expressions for the K.E and P.E of a vibrating stretched string and hence find its total energy. (6+2)

b) In a musical instrument two strings are of the same length and material and have diameters in the ratio 6:1. They are to be stretched so that one string gives a note whose frequency is 3 times that of the other string. Calculate the stretching force of the second string if the stretching force for the first string is 2 Kg wt. (2)

Answer **<u>any one</u>** question of the following:

14. Show that in steady state, the average power spent by the external periodic force is equal to the rate of work done against the damping. (5)

15. a) Show that in forced vibration $\frac{\text{average kinetic energy}}{\text{average potential energy}} = \frac{\omega^2}{\omega_0^2}$, where ω_0 is the natural frequency of the oscillator and ω is the frequency of the forcing system. (4)

b) Why is damping force usually taken to be proportional to instantaneous velocity? (1)

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